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**Mathematics: analysis and approaches**  
**Higher level**  
**Paper 1**

Friday 6 May 2022 (afternoon)

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

### Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Find the value of  $\int_1^9 \left( \frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$ .

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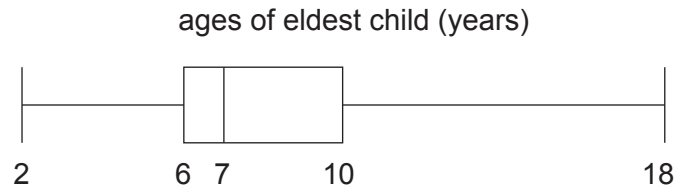


2. [Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult,  $a$  years old, and of their eldest child,  $c$  years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale



(a) Find the largest value of  $c$  that would not be considered an outlier. [3]

The regression line of  $a$  on  $c$  is  $a = \frac{7}{4}c + 20$ . The regression line of  $c$  on  $a$  is  $c = \frac{1}{2}a - 9$ .

(b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.

(ii) Find the mean age of all the adults surveyed. [4]

A large rectangular area containing horizontal dotted lines for writing answers.



3. [Maximum mark: 7]

Consider the functions  $f(x) = \sqrt{3} \sin x + \cos x$  where  $0 \leq x \leq \pi$  and  $g(x) = 2x$  where  $x \in \mathbb{R}$ .

(a) Find  $(f \circ g)(x)$ . [2]

(b) Solve the equation  $(f \circ g)(x) = 2 \cos 2x$  where  $0 \leq x \leq \pi$ . [5]

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4. [Maximum mark: 5]

Consider the curve with equation  $y = (2x - 1)e^{kx}$ , where  $x \in \mathbb{R}$  and  $k \in \mathbb{Q}$ .

The tangent to the curve at the point where  $x = 1$  is parallel to the line  $y = 5e^kx$ .

Find the value of  $k$ .

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5. [Maximum mark: 7]

Consider  $f(x) = 4 \sin x + 2.5$  and  $g(x) = 4 \sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$ , where  $x \in \mathbb{R}$  and  $q > 0$ .

The graph of  $g$  is obtained by two transformations of the graph of  $f$ .

(a) Describe these two transformations. [2]

The  $y$ -intercept of the graph of  $g$  is at  $(0, r)$ .

(b) Given that  $g(x) \geq 7$ , find the smallest value of  $r$ . [5]

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6. [Maximum mark: 5]

Consider the expansion of  $\left(8x^3 - \frac{1}{2x}\right)^n$  where  $n \in \mathbb{Z}^+$ . Determine all possible values of  $n$  for which the expansion has a non-zero constant term.

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12EP07

Turn over



7. [Maximum mark: 8]

The continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{k}{\sqrt{4-3x^2}}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of  $k$ .

[4]

(b) Find  $E(X)$ .

[4]

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8. [Maximum mark: 6]

Consider integers  $a$  and  $b$  such that  $a^2 + b^2$  is exactly divisible by 4. Prove by contradiction that  $a$  and  $b$  cannot both be odd.

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12EP09

Turn over

9. [Maximum mark: 6]

Consider the complex numbers  $z_1 = 1 + bi$  and  $z_2 = (1 - b^2) - 2bi$ , where  $b \in \mathbb{R}$ ,  $b \neq 0$ .

(a) Find an expression for  $z_1z_2$  in terms of  $b$ . [3]

(b) Hence, given that  $\arg(z_1z_2) = \frac{\pi}{4}$ , find the value of  $b$ . [3]

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12EP10

Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

Consider the series  $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$ , where  $x \in \mathbb{R}, x > 1$  and  $p \in \mathbb{R}, p \neq 0$ .

(a) Consider the case where the series is geometric.

(i) Show that  $p = \pm \frac{1}{\sqrt{3}}$ .

(ii) Hence or otherwise, show that the series is convergent.

(iii) Given that  $p > 0$  and  $S_{\infty} = 3 + \sqrt{3}$ , find the value of  $x$ . [6]

(b) Now consider the case where the series is arithmetic with common difference  $d$ .

(i) Show that  $p = \frac{2}{3}$ .

(ii) Write down  $d$  in the form  $k \ln x$ , where  $k \in \mathbb{Q}$ .

(iii) The sum of the first  $n$  terms of the series is  $\ln\left(\frac{1}{x^3}\right)$ .

Find the value of  $n$ . [12]

11. [Maximum mark: 15]

Consider the three planes

$$\Pi_1 : 2x - y + z = 4$$

$$\Pi_2 : x - 2y + 3z = 5$$

$$\Pi_3 : -9x + 3y - 2z = 32$$

(a) Show that the three planes do not intersect. [4]

(b) (i) Verify that the point  $P(1, -2, 0)$  lies on both  $\Pi_1$  and  $\Pi_2$ .

(ii) Find a vector equation of  $L$ , the line of intersection of  $\Pi_1$  and  $\Pi_2$ . [5]

(c) Find the distance between  $L$  and  $\Pi_3$ . [6]



Do **not** write solutions on this page.

**12.** [Maximum mark: 21]

The function  $f$  is defined by  $f(x) = e^x \sin x$ , where  $x \in \mathbb{R}$ .

(a) Find the Maclaurin series for  $f(x)$  up to and including the  $x^3$  term. [4]

(b) Hence, find an approximate value for  $\int_0^1 e^{x^2} \sin(x^2) dx$ . [4]

The function  $g$  is defined by  $g(x) = e^x \cos x$ , where  $x \in \mathbb{R}$ .

(c) (i) Show that  $g(x)$  satisfies the equation  $g''(x) = 2(g'(x) - g(x))$ .

(ii) Hence, deduce that  $g^{(4)}(x) = 2(g'''(x) - g''(x))$ . [5]

(d) Using the result from part (c), find the Maclaurin series for  $g(x)$  up to and including the  $x^4$  term. [5]

(e) Hence, or otherwise, determine the value of  $\lim_{x \rightarrow 0} \frac{e^x \cos x - 1 - x}{x^3}$ . [3]

**References:**

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12EP12